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Verein Deutscher Ingenieure
Deutsche Gesellschaft für Qualität

Statistical Testing of the Operational and Positional Accuracy of Machine Tools
Basis

VDI/DGQ 3441

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Contents

VDI-Gesellschaft Produktionstechnik (ADB)
Fachbereich Produktionsanlagen
(VDI Production Engineering Group (ADB)
Technical Department for Production Plant

Foreword

Since the beginning of the sixties two completely independent working parties have been engaged on the investigation of statistical test methods for machine tools. On the one hand a group of the Deutsche Gesellschaft für Qualität (DGQ) (German Quality Association) has been concentrating on statistical testing with the aid of test workpieces, while the Ausschuß Informationsverarbeitung der VDI-Gesellschaft Produktionstechnik (ADB) (Data processing Committee of the Production Engineering Section of the VDI) has been working mainly on problems connected with the positional accuracy of NC machine tools. The work of both groups has been embodied in the following publications:

ASQ/AWF 12	Statistical Testing of the Operational Accuracy of Machine Tools
ASQ/AWF 12–1	Statistical Testing of the Operational Accuracy of Lathes
VDI 3254 section 1	Numerically Controlled Machine Tools; Accuracy Data; Terms and Static Characteristics
section 2	–; –; Measurement of Static Characteristics
section 3	–; –; Testing with Test Workpieces

The two groups were amalgamated in 1970 with the VDI/DGQ Fachgruppe Statistische Prüfung der Arbeitsgenauigkeit von Werkzeugmaschinen (Technical Group for the Statistical Testing of the Accuracy of Machine Tools) supported jointly by the VDI and DGQ, and which is in addition actively sustained and promoted by the Verein Deutscher Werkzeugmaschinenfabriken e. V. (VDW) im VDMA (German Association of Machine Tool Manufacturers) and the Fachnormenausschuß Werkzeugmaschinen (FWM) (Machine Tools Engineering Standards Committee). It was agreed that the DGQ would be in charge and the results of the work would be published as VDI/DGO guidelines. The earlier publications were thoroughly revised and extended.

The individual guidelines have the following titles:

VDI/DGQ 3441	Statistical testing of the operational and positional accuracy of machine tools; basic principles
VDI/DGQ 3442	Statistical testing of the operational accuracy of lathes
VDI/DGQ 3443	Statistical testing of the operational accuracy of milling machines
VDI/DGQ 3444	Statistical testing of the operational accuracy of drilling machines
VDI/DGQ 3445 Sections 1 to 5 *)	Statistical testing of the operational accuracy of grinding machines

*) Fundamentals of: cylindrical grinding machines with centres; centreless, internal and surface grinding machines.

Statistical test methods may of course be used for other types of machine tool not dealt with in the above-mentioned guidelines. The conditions of operation and testing must be established and agreed for any particular case.

The investigations were directed by Dr.rer.nat. *Klaus G. Müller*, Stuttgart (Chairman of the VDI/DGQ Technical Group). The individual Specifications were drawn up in working parties consisting of delegates from manufacturers and users in addition to the associations. Thanks are due to all chairmen, collaborators and companies involved for their willing co-operation and support.

The following assisted in the preparation of the guideline VDI/DGQ 3441:

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Dipl.-Ing. *Buck*, Stuttgart
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Dipl.-Ing. *Golücke,* Aachen
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Dipl.-Ing. *Schäfer*, Stuttgart
Dr.-Ing. *Schmid,* Nürtingen

Preliminary Note

The accuracy of a machine tool is in general assessed in accordance with the standards of the DIN-Deutsches Institut für Normung e. V.[1]).

Testing in accordance with these conditions of acceptance serves mainly to provide information concerning the geometrical accuracy of machine tools determined as a rule with the machine in the unloaded state. The effects of, for example, cutting forces, play in bearings and imbalance which arise during machining of workpieces, are under these circumstances only partly comprehended.

General specifications for the acceptance of machine tools are contained in DIN 8601.

Besides the tests described in the standards there are other factors which are important when assessing the quality of a machine tool.

Finally, the quality of the components to be machined is decisive for the *operational accuracy* of a machine. Only statistical methods are able to provide reliable evidence of this. Furthermore, the determination of *positional accuracy* is particularly important, especially in the case of numerically controlled machine tools. Statistical methods are also used for this purpose. Acceptance in accordance with the standards of the DIN Deutsches Insitut für Normung e. V. (German Standards

1) Information about existing German and international standards is available from the Fachnormenausschuß Werkzeugmaschinen (FWM), (Machine Tools Engineering Standards Committee), Frankfurt/M.

Institution) is not rendered superfluous by the use of these procedures, but should in each case precede the statistical test.

1 Scope

This guideline describes the basic principles of the statistical test methods. It is of direct application to the testing of machines which are *tied to a particular component*, that is for all special machines where the parts to be machined are clearly defined. The Specification can also be used in its general sense for all machines which are *not tied to a particular component*, length measuring gauges, as well as for coordinate measuring machines, component and coordinate tables, and also for copying equipment and drawing machines. Furthermore, it forms the basis for an understanding of the guidelines VDI/DGQ 3442 to 3445, in which the method of checking the operational accuracy of lathes and milling, drilling and grinding machines which are not tied to a particular component is explained in more detail.

All these guidelines are recommendations. It is a matter for agreement between manufacturer and customer whether they are to be taken as the basis for the acceptance of new machines. Furthermore, the results of such tests can provide basic information to assist in production planning and ensure reliability of manufacture. The tests described are suitable for checking the operation of machines which have been in use for a long period, or they can be directed to the analysis of weak points indicated by the occurence of defective parts.

2 Introduction

In this guideline the terms, procedures and methods of statistical testing are clearly defined and described, not only for the operational accuracy of machine tools but also for their positional accuracy. It specifies the characteristics to be measured and at the same time gives recommendations for their uniform presentation.

In accordance with the usage recommended in standards, e.g. to express the accuracy of a piece of measuring equipment directly by the magnitude of its measurement uncertainty, the following terms have been adopted in this publication

manufacturing uncertainty as a measure of manufacturing accuracy,

operational uncertainty as a measure of the operational accuracy

and

positional uncertainty as a measure of the positional accuracy.

In the first place dimensional variations which arise in the machining of components give a direct indication of manufacturing scatter or accuracy. The manufacturing uncertainty is therefore a measure of the accuracy with which a component can be produced on a given machine under specific conditions of operation. It includes deviations which are attributable to the machine – by definition the operational uncertainty of a machine tool – and deviations which are not attributable to the machine. Since for this present assessment only those deviations which are attributable to the machine are of interest, these must be identified from the large number of possible factors which have an effect on the manufacturing uncertainty, Fig. 1.

It is not the object of this Standard to investigate the manufacturing uncertainty using all the factors outlined in Fig. 1.

By definition all deviations which are attributable to the machine which occur during the production of parts on a machine tool are included in the term *operational uncertainty*, so that it not only embraces defects in the machine system but also random defects, Fig. 2.

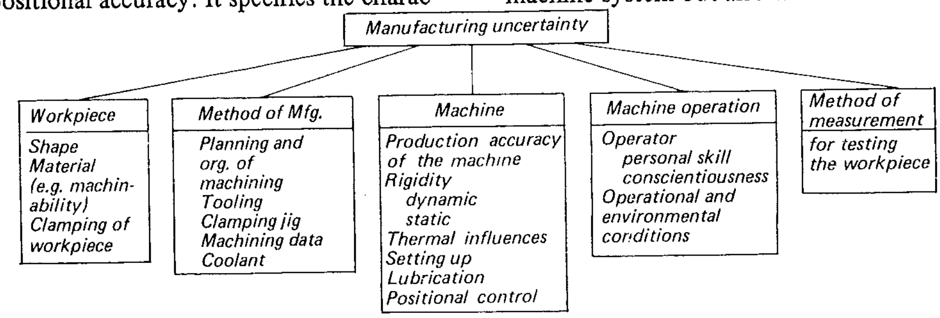


Fig. 1. Factors affecting the manufacturing uncertainty of a workpiece.

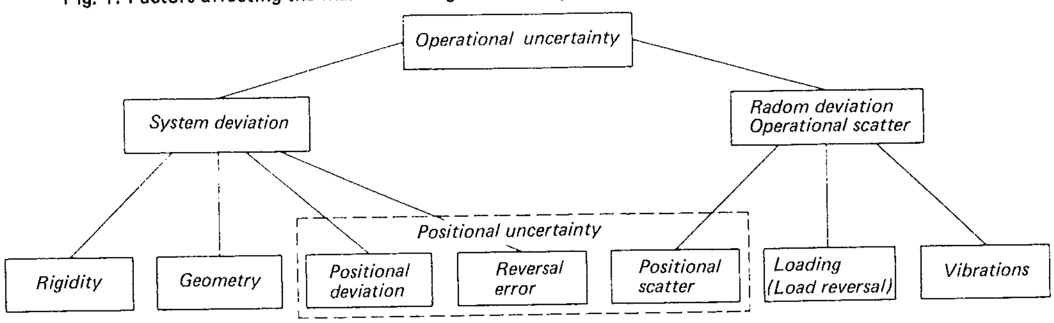


Fig. 2. Operational uncertainty of the machine-limiting factors

It is not at present possible to devise a concise direct test method for determining the operational uncertainty of a machine tool. An indication of the most important factors attributable to the machine can only be given using various indirect tests.

The detection of defects of the machine system affecting operational uncertainty is a complex problem which can be very difficult. The DIN Standards and the corresponding ISO Recommendations form important starting points for the purpose of geometrical tests.

Furthermore, special investigations for instance into the dynamic behaviour of a machine tool can effect progress.

Operational scatter is a measure of all random deviations attributable to the machine. It is an indication of the degree of reproducibility to which a component can be produced on a machine tool. By machining test workpieces under clearly defined machining conditions the operational scatter can be determined by the statistical methods in accordance with section 3.

Positional accuracy is an important characteristic feature in the case of all machines having a facility for positioning, especially where numerically controlled machines are concerned. Its magnitude is defined as *positional uncertainty* (Fig. 2). Positional uncertainty indicates the degree of accuracy with which an arbitrarily preselected position can be reached or started from. It is determined by direct length measurements on the machine in accordance with the procedures described in section 4.

All deviations are detected in this way step by step, after the saddle of a machine tool for instance comes to rest in the various positions along the axis of test. Testing the dynamic behaviour of a machining axis is not dealt with in this guideline.

3 Testing Operational Accuracy by Machining Test Workpieces

Besides other characteristics which are taken into account when assessing the quality of a machine tool, it is usual to consider the accuracy with which a part can be produced on the machine. One speaks of the operational accuracy of a machine tool. The measure of operational accuracy is defined as *operational uncertainty.*

3.1 Operational Uncertainty

Operational uncertainty is the sum of all random deviations and those which are attributable to the machine system determined for example by the use of test workpieces (Fig. 2).

It is determined for a specific method of production and under defined conditions of manufacture.

Dimensional variations of the part which occur during machining give the first indication of manufacturing uncertainty (Fig. 1).

By the establishment of test conditions and the use of statistical methods those influences which are not attributable to the machine (e.g. trend due to tool wear, changes of operator or measurement uncertainty when measuring individual parts) can be eliminated from the test data, in order to determine the magnitude of the operational uncertainty.

In most instances the expense of determining the operational uncertainty is not economically justified. For this reason only the share of random deviations which can be attributed to the machine are determined, i.e. the operational scatter, using test workpieces.

3.1.1 Deviations Due to the Machine System

By these are meant effects such as geometrical deviations of the machine components, thermal effects, static and dynamic rigidity etc. They can to some extent be detected for instance by geometrical checks in accordance with DIN Standards and ISO Recommendations or by special investigations (dynamic behaviour). By laying down definite machining conditions and the use of the statistical methods described later they can be eliminated to a great extent in the same way as for effects which are not attributable to the machine.

3.1.2 Random Deviations – Operational Scatter

The operational scatter of a machine tool can be determined from the dimensional variations of the test workpieces under specific machining conditions using statistical methods. The operational scatter is defined as six times the standard deviation ($6 \cdot s$) determined from the dimensional variations of the test workpieces (section 3.2.4). It is always stated in conjunction with the type of deviation (size, position, form).

3.2 Determination of the Operational Scatter A_s

The procedures described in this section are applicable without difficulty to machines which are tied to a particular component, and in the case of all special machines which by virtue of their design and the fact that they are custom built (and possibly the machining conditions), the parts to be machined are already defined. The test can then also be carried out using such parts as are produced on the machine.

These procedures are likewise applicable to all machines which are not tied to a particular component. It is, however, necessary to specify the requisite typical test workpieces and machining conditions such as for example cutting speed, feed rates and operating conditions for the different types of machine tool. Details are set out in the special guidelines VDI/DGQ 3442 to 3445.

3.2.1 Machining of Test Workpieces

The machining of test workpieces enables all the essential characteristics to be determined. In the case of machines not tied to a particular component, suitable test workpieces or workpieces which are similar to one another taken from current production are necessary. Their dimensional proportions should be as far as possible constant.

Apart from dimensional deviation every workpiece exhibits other deviations from the ideal shape which are composed of individual geometrical elements. Deviations of shape are defined in DIN 7148. It depends on the method or purpose of production which of the possible deviations which occur in test workpieces shall be investigated in more detail.

3.2.1.1 Preparatory Arrangements

Before beginning the testing of a machine tool by machining test workpieces the following must be settled in order to ensure that it is arranged to give positive results at reasonable cost:

1. Allowable deviation of size, position or shape;
2. Technical conditions, tools and operation layout;
3. Number of test workpieces;
4. Limiting conditions (e.g. setting up the machine, environment);
5. Arrangements for measuring the test workpieces.

Only when these data are known is it possible to analyse the test results and so determine the operational scatter satisfactorily.

It is very important to ensure that the environmental conditions are comparable when comparing the test results of several machine tools of the same type or of one machine, the change in the operational scatter of which it is intended to follow up over an extended period.

Random deviations can only be determined statistically, for which it is necessary to machine at least 25 to 50 components. Using statistical methods it is also possible to separate and eliminate from the results machine systems effects such as trend from the random deviations.

3.2.2 Methods of Test

There is a choice of two test procedures, the standard test and the short test. These are distinguished mainly by the type of random sampling.

3.2.2.1 Standard Test

As tool wear increases there is a change on the one hand of the operational scatter of a machine tool due to increased loading brought about by higher cutting forces, while on the other hand the deviation due to the machine system also changes e.g. as regards size (trend). It is therefore important that the random sampling covers the whole of the running time between two tool changes. The test is begun with sharpened tools[2]).
Random samples are taken over the expected tool life.

In the case of *single spindle machines* 10 individual random samples are generally taken for every 5 parts.

[2]) High initial wear on the cutter teeth can falsify the test results considerably. For this reason, when using newly sharpened tools or new throw away tips a few cuts should be made prior to the test.

In the case of *multi-spindle-machines* or transfer lines where similar parts are machined on several spindles or work carriers, separate checks must be made for each spindle and locating station in order to detect every weak point. Generally 10 random samples are taken for every 3 workpieces from each spindle or work carrier. The measured values are analysed in accordance with the range method (see section 6).

3.2.2.2 Short Test

If a quick overall picture of the operational scatter is required and the costs of investigation must be kept as low as possible, short tests may be sufficient, for example when machining with a single spindle machine with sharpened tools[2]), as far as possible 50 parts are run off one after the other.

If one has to make do with a smaller number of pieces (not less than 10) the test may be sufficient, but the information yielded will be less reliable (see section 6.3).

The measured values are generally analysed in accordance with the range method in section 6. Analysis using the probability diagram can only be made if the effect of trend is not present.

Compared with the standard test, the short test offers quicker information, a relatively simple procedure and little expense. It should be borne in mind, however, that the tools can machine many more components than the number of pieces tested. This means that the actual magnitude of the operational scatter which occurs due to the greater machining forces operating as the tools become blunted, is not fully measured. The short test can therefore only be regarded as an approximate method. It is not representative of the total operational scatter. The standard test is to be preferred. On grounds of cost the short test is used mainly for testing machine tools which are not tied to a particular component.

3.2.3 Checking the Dimensions

The dimensions can be checked in two ways:

by random selection of the measuring position;

by precise measurement of the smallest and greatest dimension.

3.2.3.1 Checking the Dimensions by Random Selection of the Measuring Position

The dimension is checked at random at arbitrary positions on, for example, a turned part. The procedure is outlined in Fig. 3. The measured values summate the dimensional and shape deviations relating to circular and cylindrical shape. These dimensional and shape deviations cannot be calculated separately from the measured values thus determined. Dimensional checking by random selection of the measuring position will therefore always be chosen if the expected shape deviations are so small relative to the operational scatter that they can be neglected.

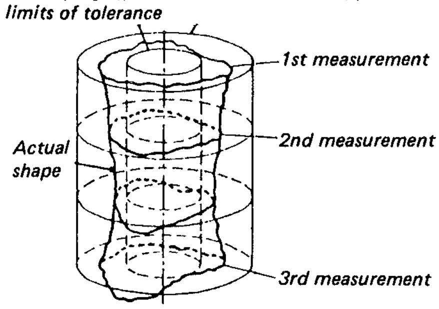


Fig. 3. Random dimensional checking

3.2.3.2 Checking the dimensions by precise measurement of the smallest and greatest dimension, Fig. 4

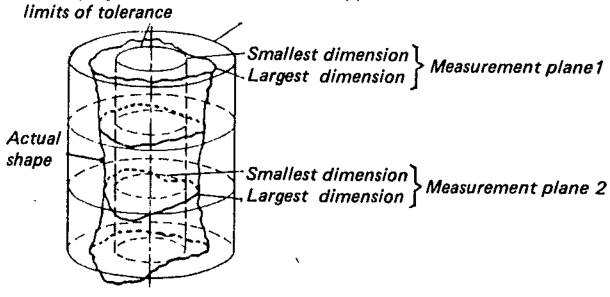


Fig. 4. Checking the greatest and smallest dimension

If the expected shape deviations are too great relative to the operational scatter, the operational scatter of the dimensions and – if required – the shape deviations must be separately determined, since otherwise the operational scatter of the machine with reference to size would be falsified due to the effect of shape defects.

The influence of defects of shape are eliminated by precise measurement either of the smallest or the greatest dimension of the test workpiece. The measured values are analysed in accordance with section 3.2.4.

If required, the shape deviation, for example eccentricity, is separately measured and analysed statistically.

3.2.4 Formulas for the Determination of Operational Scatter A_s

When carrying out a standard test of a machine in accordance with section 3.2.2.1 $m = 10$ random samples for every 5 parts are taken, distributed over a complete period of tool life. The measured values are recorded in the test record (figures 5a and b) in the order of their machining. 1 random sample is made up of every $n = 5$ parts in succession.

Formulas for the determination of operational scatter in accordance with the range method:

Term	Formula
Range R in the random sample j	$R_j = x_{i\,max} - x_{i\,min}$
Mean range	$\bar{R} = \frac{\sum_{j=1}^{m} R_j}{m}$
Standard deviation	$s_R = \frac{\bar{R}}{d_n{}^{*)}}$
Operational scatter	$A_s = 6 \cdot s_R$

*) d_n is a factor which depends on the number of measured values in the random sample (section 6.2.2).

The operational scatter A_s which is worked out in this way for the machine indicates that – under the specific operational conditions laid down – 99.7 % ($6 \cdot s$ range) of the random dimensional scatter attributable to the machine are smaller than or equal to this value for A_s.

3.3 Operational Scatter and Workpiece Tolerance

The dimensional variations of the machined parts are brought about by different causes (section 2)

The principal causes are:

operational scatter,

adjustment and measurement uncertainty,

trend, e.g. tool wear, temperature.

The statistical test described here is intended to establish whether, with a given material and given operational conditions, the dimensional scatter of the machined parts caused by the machine lies within the manufacturing tolerance. The operational scatter determined by the test must therefore not exceed a specific quantity as regards workpiece tolerance. The relationship of operational scatter to tolerance is designated relative operational scatter f

$$f = \frac{\text{operational scatter } A_s}{\text{manufacturing tolerance } T} \cdot 100\,\%$$

In the case of chip forming machining where the tool wear only causes a small dimensional change, an operational scatter of up to 80 % of the workpiece tolerance may be allowed, for instance when drilling. In the case of machining processes *with trend* the operational scatter should not exceed 60 % of the workpiece tolerance. The following percentage distribution of the workpiece tolerance gives an approximate summary of the situation:

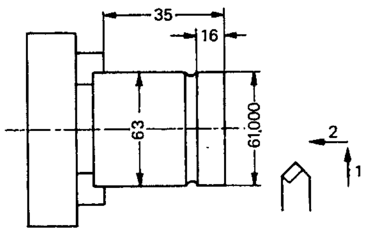
	with trend	without trend
relative operational scatter f	60 % of tolerance	80 % of tolerance
trend	20 % of tolerance	– –
measurement uncertainty + adjustment uncertainty	20 % of tolerance	20 % of tolerance

VDI/DGQ 3441	Determination of the operational accuracy of a lathe (to VDI/DGQ 3442)	Test record page 1

Machine: Leit- und Zugspindeldrehmaschine
Manufacturer: Schulz u. Co   Type: DLZ 300
Machine number: 1027   Year of manufacture: 1967
Swing over bed: 300 mm   Last overhaul: Mai 1975
Clamping: Dreibackenfutter 250 ∅   Foundation: lose aufgestellt auf St.-Platten
Positioning device: Handrad mit Skalenring
Testing place: W31
Remarks:

Test workpiece:
Workshop: C45
Dimensions
Diameter D = 63 mm
Length l = 35 mm
Gauge length l_1 = 16 mm
Standard reference dimension D_1 = 61,000 mm
Tolerance +20 µm
Remarks:

Working drawing



Cutting conditions
Standard values to VDI/DGQ 3441 ☒   Deviating values ☐
RPM n = 560 U/min   Cutting speed v = 106 m/min
Depth of cut a = 0,5 mm   Feed s = 0,1 mm/U (mm/min)
Remarks

Tooling: Type of carbide: P10
Cutting geometry: adjustable rake = 50°   Radius of point r = 0,5 mm
orthogonal rake = 8°   Back rake = 9°
clearance angle = 6°
Short designation:

Measuring gauge (designation): Meßschraube mit Feinzeiger
Scale reading: 1 µm   Location with: Endmaßen   Date: 6.6.75
Measurement uncertainty: (cf VDI/DGQ 3441 section 6.4)
$4\,s_{R\,equipt} \leq 0{,}2$ Tolerance ☒ (1)
$> 0{,}2$ Tolerance ☐ (2)   Checked: V.C.
In case(2) $s_{R\,equipt}$ must be considered in relation to the actual machine scatter according to ⑯ in the test record (page 2)

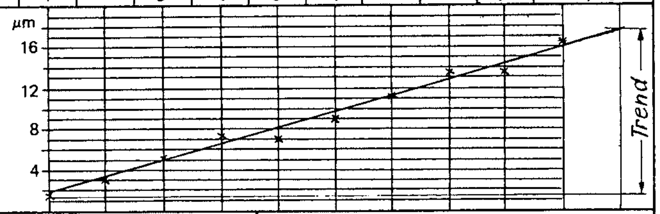
Fig. 5a. Test record page 1

VDI/DGQ 3441	Determination of operational accuracy	Test record page 2

Tabular values in µm as deviations from the set dimension: 61,000 mm

① Clamping position / random sample j / testpiece i	I	II	III	IV	V						
	1	2	3	4	5	6	7	8	9	10	
1	1	2	4	5	6	7	9	11	10	14	
2	0	2	3	6	5	6	8	10	11•	13	
3	1	3	5	9	6	10	12	13	13	16	
4	2	2	6	6	7	9	11	14	15	17	
5	4	6	8	10	11	13	16	19	18	22	
② Σx_i	8	15	26	36	35	45	56	67	67	82	Σ
③ $\bar{x}_j = \Sigma x_i / n$	1,6	3	5,2	7,2	7	9	11,2	13,4	13,4	16,4	87,4
④ $R_j = x_{i\,max} - x_{i\,min}$	4	4	5	5	6	7	8	9	8	9	65

⑤ $\bar{x}$-chart



Analysis

⑥ Total mean value $\bar{\bar{x}} = \frac{\Sigma \bar{x}_j}{m} = \frac{87{,}4}{10} = 8{,}7$ µm

⑦ Range mean value $\bar{R} = \frac{\Sigma R_j}{m} = \frac{65}{10} = 6{,}5$ µm

⑧ Standard deviation $s_R^{**} = \frac{\bar{R}}{d_n} = \frac{6{,}5}{2{,}326} = 2{,}8$ µm (d_n = 2,326 bei n = 5)

Trend: (as far as necessary)

⑨ Number of pieces machined N = 50 Stck.

⑩ T_N (from $\bar{x}$-chart) = 15 µm

⑪ T_s (trend per piece) $= \frac{T_N}{N-1} = \frac{15}{49} = 0{,}31$ µm

⑫ Allowance for trend (VDI/DGQ 3441, section 6.5.2.2)

i \ j	1	2	3	4	5	6	7	8	9	10	
1	1	0,45	0,9	0,35	-0,2	-0,75	-0,3	0,15	-2,4	0,05	
2	-0,31	0,14	-0,41	1,04	-1,51	-2,06	-1,61	-1,16	-1,71	-1,26	
3	0,38	0,83	1,28	3,73	-0,82	1,63	2,08	1,53	-0,02	1,43	
4	1,07	-0,48	1,97	0,42	-0,13	0,32	0,77	2,22	1,67	2,12	$\Sigma R_{j\,korr}$
5	2,76	3,21	3,66	4,11	3,56	4,01	5,46	6,91	4,36	6,81	
$R_{j\,korr}$	3,07	3,69	4,07	3,76	5,07	6,07	7,07	8,07	6,76	8,07	55,7

⑬ $\bar{R}_{korr} = \frac{\Sigma R_{j\,korr}}{m} = \frac{55{,}7}{10} = 5{,}6$ µm

⑭ $s_R^{*} = \frac{\bar{R}_{korr}}{d_n} = \frac{5{,}6}{2{,}326} = 2{,}4$ µm

Regard to measurement uncertainty (as necessary to VDI/DGQ 3441, section 6.4)

⑮ $s_{R\,equipt}$ = 0,9 µm

⑯ $s_R = \sqrt{s_R^{*2} - s_{R\,equipt}^2} = 2{,}2$ µm (s_R taken from ⑧ or ⑭ as appropriate)

⑰ Operational scatter $A_s = 6 \cdot s_R = 13{,}2$ µm (s_R taken from ⑧, ⑭ or ⑯ as appropriate)

Date: 21.6.76   Signature: Teichmann

Fig. 5b. Test record page 2

This means, assuming that no trend effect is present (i.e. the operational scatter may not amount to 80 % of the given workpiece tolerance), that the mean *range* $\bar{R}$, formed from groups of 5 measured values, may only take up about 30 % of the given workpiece tolerance! This is given by the following relationship:

$$0.8\,T = 6\,s_R = 6 \cdot \frac{\bar{R}}{d_n}; (d_n \text{ for 5 test workpieces} = 2.33)$$

from which

$$\underline{\bar{R} \approx 0.3\,T}$$

3.4 Correction Factors – Trend, Measurement Uncertainty

The magnitude of operational scatter is very much influenced by the factors of trend and measurement uncertainty.

3.4.1 Trend

The results will be unduly falsified if the trend – e.g. changes of the machined dimension due to tool wear or thermal effects – is not eliminated by calculating the operational scatter from the measured values.

The effect of trend on the standard deviation which is being calculated is already largely eliminated by determining the operational scatter in accordance with the range method described in section 6.2. If, however, the slope of the trend line is too steep, the standard deviation should be corrected and the operational scatter calculated from it in accordance with section 6.5.2.

Knowledge of the magnitude and course of the trend T_N yields important information about the effects of systems errors on the machining process.

3.4.2 Measurement Uncertainty

The measuring gauges used to determine the measured values must be sufficiently accurate. It is therefore necessary to specify the measurement uncertainty of the measuring gauges and to relate this to the workpiece tolerance or operational scatter. If the resulting measurement uncertainty $4\,s_{R\ \text{equipt.}}$ is too high, more accurate measuring gauges must be used or the measurement uncertainty must be considered mathematically. For the determination and consideration of measurement uncertainty see section 6.4.

3.5 Test Record with Example of the Determination of the Operational Scatter of a Machine

The test record used for determining the operational scatter of machines by machining workpieces is shown in Fig. 5a and b. Page 1 (Fig. 5a) is arranged for a specific machine, and contains all the necessary data concerning the machine itself, the test workpieces, the machining conditions and information about the measuring gauges used to check the dimensions.

Page 2 (Fig. 5b) of the test record is arranged to cover all types of machine in a general way. The measured values – which were determined from the test workpieces – are recorded in section ①, and the method of calculating operational scatter is set out in lines ② to ⑰.

To give a clearer appreciation of this an example is given of the calculation of the operational scatter A_s of a lathe.

The test workpiece is a turned part. The diameter is machined to a nominal size of 61.0 mm. The machining tolerance is 20 μm. All the necessary data for assessing the results (material, machining conditions) are recorded on page 1 (Fig. 5a) of the test record.

The measured values are recorded in the order of machining in section ① of page 2 of the test record. There are $n = 5$ successive individual values for one random sample m (recorded in one column).

The average values $\bar{x}_j$ worked out for each random sample, line ③, are plotted on the $\bar{x}$ chart; line ⑤. Deviations of the system, for example trend $T_N = 15$ μm, can be recognised from the $\bar{x}$ chart for 50 machined parts. The aggregate of the mean values $\bar{\bar{x}} = 8.7$ μm is obtained from the $\bar{x}$ values; line ⑥.

The approximate standard deviation determined by the range method is $s_R^{**} = 2.8$ μm; line ⑧. This standard deviation s_R^{**} includes not only the effect of trend but also the measurement uncertainty of the measuring procedure. If these two factors are too high relative to the tolerance their effect must be eliminated theoretically. In the example shown both corrections were made for the sake of completeness.

For a number $N = 50$ machined parts, the trend $T_N = 15$ μm, obtained from the $\bar{x}$ chart line ⑤; ⑨, ⑩. The mean corrected range $\bar{R}_{corr}$ ⑬ works out at 5.57 μm calculated by the method given in section 6.5. It should be noted that in order to calculate the part of the trend x_{iT} appertaining to each individual measured value, all the measured values must be numbered consecutively from $i = 1$ to $i = 50$:

$$x_{iT} = (i - 1)\,T_s \text{ where } i = 1 \text{ to } 50.$$

The standard deviation s_R^{*} adjusted for the effect of trend is calculated as shown in line ⑭, and still contains the influence of the measurement uncertainty of the measuring procedure.

The standard deviation of the measurement uncertainty $s_{R\ \text{equipt}}$ ⑮ was calculated as 0.9 μm using the method described in section 6.4. Putting this in the equation in line ⑯ the standard deviation s_R becomes 2.2 μm. The operational scatter of the machine $A_s = 6\,s_R$ then becomes 13.2 μm; line ⑰.

4 Checking Positional Accuracy by Direct Measurement on the Machine

4.1 Factors in the Assessment of Positional Accuracy

Factors in the assessment of the accuracy with which a position can be attained with machining and testing equipment are

positional tolerance,

positional uncertainty,

positional deviation,

reversal error,

positional scatter.

These factors are separately determined for each axis with the machine in the unloaded state. Positional accuracy within the plane and space of the machining and testing equipment depends in addition on orthogonality and linearity.

4.1.1 Positional Tolerance T_p

Positional tolerance T_p represents the total allowable deviation within the working range of a machine axis. The positional uncertainties determined for the individual axes of the equipment must be less than or equal to the manufacturer's quoted positional tolerances for the equipment.

4.1.2 Positional Uncertainty P

The positional uncertainty P is the total deviation on the selected test axis taking into account the characteristic values determined in the individual positions:

Positional deviation, reversal error, and positional scatter.

It consequently includes both systems and random deviations.

4.1.3 Positional Deviation P_a

The positional deviation P_a as the system deviation is the maximum difference of the mean values of all measuring positions on a selected test axis.

4.1.4 Reversal Error U

Reversal error as a system deviation represents the difference obtained from the mean values in both directions of travel for each position on the chosen test axis.

The mean reversal error $\overline{U}$ is the arithmetic mean value of the reversal error of all measuring positions on the chosen test axis.

4.1.5 Positional scatter P_s

The positional scatter P_s represents the effect of random deviations in each position on the chosen test axis. It is expressed to a specified degree of probability (sections 4.2.2 and 4.2.3).

The mean positional scatter $\overline{P}_s$ is the arithmetic mean of the positional scatter of all measuring positions on the chosen test axis.

4.2 Determination of the Characteristic Values

4.2.1 Determination of the Characteristic Values of Individual Measuring Positions within the Chosen Test Axis

To determine the characteristic values, e.g. of a numerically controlled axis, a number m of positions are chosen along the whole length of travel of the test axis. From a statistical point of view this means m random samples with the serial index "j" (section 6). The measuring positions should be chosen at unequal spacings from each other so as to ensure the detection also of recurrent errors.

Each position is brought up several times, generally in opposite directions parallel to the axis. In the case of indirect position sensing devices a definite reference point must be come up to prior to each measuring cycle. The number n of individual measured values obtained in this way for each random sample is designated by the serial index "i", while the two directions of travel are marked ↑ for the positive and ↓ for the negative direction. (If the analysis of the measured results is carried out by a computer, it is suggested that the directions of travel are denoted by the abbreviations POS and NEG). In this way a number n values of x_{ij} ↑ are obtained from the positive direction and a number n values of x_{ij} ↓ from the negative direction for each random sample "j". The numerical values of the quantities x_{ij} ↑ and x_{ij} ↓ represent the deviations from the desired value at the position x_j in question. If the frequency distribution of the individual values is plotted for a random sample x_j, the resulting relationship is shown in Fig. 6.

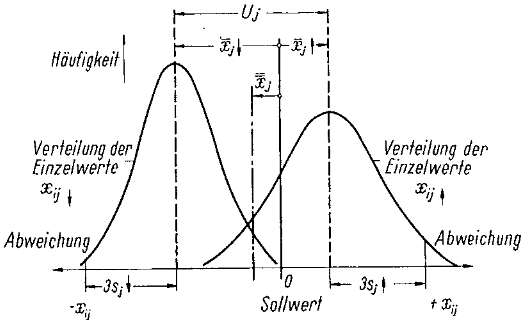


Fig. 6. Frequency distribution of the individual values x_{ij} of a measured position

The larger the number of measurement positions and the number n of individual values, the more reliable is the determination of the characteristics. The number of measurement positions to be brought up must be at least 10 per metre + 1, and, in the case of paths of travel greater than 2 metres, at least one measuring position per unit of scale. It is advisable to collate to one group at least 5 measured values at random sample position x_j from the positive and negative directions respectively.

4.2.2 Formulas for the Characteristic Values of Individual Measuring Positions within a Chosen Test Axis

To determine the individual characteristics shown in Fig. 6 and the factors defined in section 4.1 essentially simple calculations are necessary which are summarised as follows, proceeding form the individual values x_{ij}.

All values apply to a chosen test axis.

Term	Symbol
Individual value i at position x_j	x_{ij}
Individual value i at position x_j in the positive direction of travel	$x_{ij}\uparrow$
Individual value i at position x_j in the negative direction of travel	$x_{ij}\downarrow$

Term	Formulas
Mean value of the individual measured values at position x_j in the negative direction of travel	$\bar{x}_j\uparrow = \frac{1}{n}\sum_{i=1}^{n} x_{ij}\uparrow$
Mean value of the individual measured values at position x_j in the negative direction of travel of travel	$\bar{x}_j\downarrow = \frac{1}{n}\sum_{i=1}^{n} x_{ij}\downarrow$
Standard deviation of the measured values at position x_j in the negative direction of travel	$s_j\uparrow = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{ij}\uparrow - \bar{x}_j\uparrow)^2}$
Standard deviation of the measured values at position x_j in the negative direction of travel	$s_j\downarrow = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{ij}\downarrow - \bar{x}_j\downarrow)^2}$
Mean standard deviation of the measured values at position x_j	$\bar{s}_j = \frac{s_j\downarrow + s_j\uparrow}{2}$

For a quick estimation of the standard deviations of all the measured values at random sampling position x_j the range method can be used to give an approximate calculation. The ranges and the standard deviations are calculated as follows:

Term	Formulas
Range R_j at position x_j in the positive direction of travel	$R_j\uparrow = x_{ij_{max}}\uparrow - x_{ij_{min}}\uparrow$
Range R_j at position x_j in the negative direction of travel	$R_j\downarrow = x_{ij_{max}}\downarrow - x_{ij_{min}}\downarrow$
Mean range $\bar{R}_j$ at position x_j	$\bar{R}_j = \frac{R_j\uparrow + R_j\downarrow}{2}$
Mean standard deviation of the measured values at position x_j (calculated from the range)	$\bar{s}_{jR} = \frac{\bar{R}_j}{d_n{}^{*)}}$

*) d_n is a factor which depends on the number of individual measured values per random sample (section 6.2.2).

The range method procedure offers the possibility of eliminating the effect of trend on the measured values obtained in succession at random sampling position x_j (section 6.5).

4.2.3 Principles of the Characteristic Values for a Chosen Test Axis

The following principles apply to the factors defined in section 4.1:

Term	Formulas
Positional scatter at position x_j (Repeatability of position x_j, when $U_j = 0$	$P_{s_j} = 6 \cdot \bar{s}_j$
Maximum positional scatter	$P_{s_{max}} = P_{s_{j\,max}}$
Reversal error at position x_j	$U_j = \lvert \bar{x}_j\downarrow - \bar{x}_j\uparrow \rvert$
System deviation from the desired value at position x_j	$\bar{\bar{x}}_j = \frac{\bar{x}_j\uparrow + \bar{x}_j\downarrow}{2}$
Positional deviation	$P_a = \lvert \bar{\bar{x}}_{j_{max}} - \bar{\bar{x}}_{j_{min}}$
Positional uncertainty	$P = \left[\bar{\bar{x}}_j + 1/2\,(U_j + P_{sj})\right]_{max} - \left[\bar{\bar{x}}_j - 1/2\,(U_j + P_{sj})\right]_{min}$

The positional scatter P_s can only be calculated from the above formula when the random tests comprise *at least* 10 individual measured values in the positive direction of travel and 5 individual measured values in the negative direction respectively. In the exceptional case of fewer random samples, the factor K_s obtained statistically must be taken into account to determine the positional scatter (section 6.3).

If the characteristic values determined for several measured positions within the chosen test axis are plotted on the path of travel, a diagram is obtained such as is shown in Fig. 7.

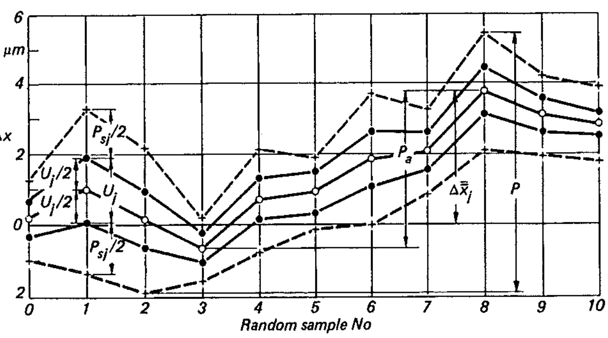


Fig. 7. Graph of the nominal sizes at several positions on the chosen axis

P	positional uncertainty
P_A	positional deviation
P_S	positional scatter
U	reversal error
x_{soll}	$x_{desired}$

4.2.4 Determination of the Mean Values of the Characteristics for the Travel Path within the Test Axes

In many cases it is sufficient to know the average values of the statistical characteristics to assess the positional accuracy. The mean values are obtained from the individual values of all the measured positions, and are therefore related to the length of the travel path within the test axis. In amplification of the principles set out in section 4.2.3 the mean values in question are given as follows:

Term	Principle
Mean positional scatter	$\bar{P}_s = \frac{1}{m} \cdot \sum_{j=1}^{m} P_{s_j}$
Mean reversal error	$\bar{U} = \frac{1}{m} \cdot \sum_{j=1}^{m} U_j$

In addition to the mean positional scatter and the mean reversal error, the extreme values $P_{s_{max}}$ and U_{max} should be stated.

4.2.5 Reference Distance and Graphical Presentation of Positional Uncertainty

The accuracy of all length measuring systems is a function of the measured length. It is therefore necessary to relate the allowable values for the characteristics to a reference distance of length L_0. If for instance the positional uncertainty is related to a length L_0 = 1000 mm, this is represented as

P (1000)

As a rule P is referred to the whole path of travel, so that there is no need to specify the reference length L_0.

If the reference length L_0 is smaller than the travel path, then the positional tolerance and the positional uncertainty must be plotted graphically in the manner described as follows to assess the accuracy.

As the distance ΔL from the reference path L_0 increases a linear rise in the positional uncertainty P is allowable. These relationships are shown diagrammatically in Fig. 8.

The cross-hatched areas can be regarded as a tolerance pattern. From Fig. 8 it can be seen that the increase in

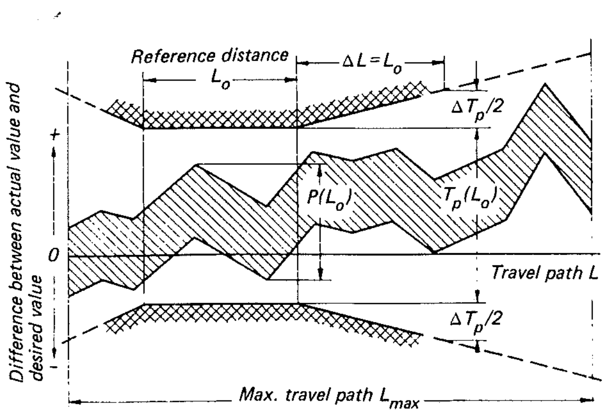


Fig. 8. Graphical presentation of positional uncertainty

positional uncertainty P is limited by the ratio $\Delta T_p/L_0$, where ΔT_p represents the increased tolerance over the path L_0. The ratio T_p/L_0 is established together with the positional tolerance T_p.

If the pattern is shifted *parallel to the axis* along the shaded area of the curves, at no point along the whole path of travel must the pattern and the curve band intersect (see Fig. 9).

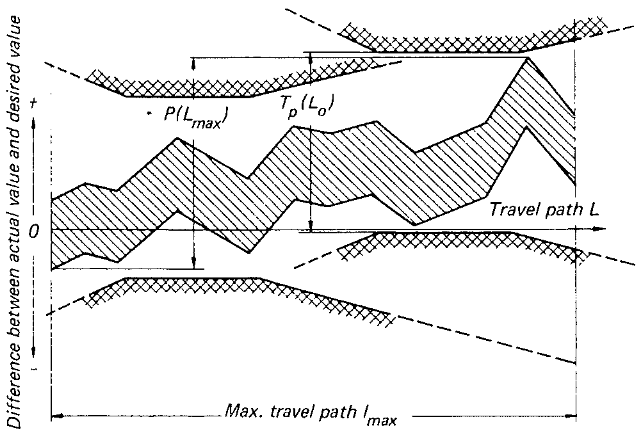


Fig. 9. Graph of a tolerance pattern shifted parallel to the axis

4.3 Obtaining the Measured Values

4.3.1 Measuring Devices and Measuring Arrangements

In order to ensure that the characteristics are measured to a sufficient degree of reliability, length measuring equipment must be used having inherent errors which are so small as to enable the allowable deviations on the machine to be detected with sufficient accuracy (section 6).

Position measurements must be carried out in the working range of the tools. They include deviations arising for instance from slideway errors and whose magnitude depends inter alia on the distance 1 between the machine measurement axis and the test axis. Fig. 10 shows the relationships for a horizontal drilling machine.

Typical positions within the working range of the tools are given for instance in the case of a drilling machine with 3 axes, if the slides on 2 axes are in a mean position, while the slide of the 3rd axis of the available slideway is measured. If limiting values are required to portray the positional accuracy, the test axes must be situated at the limits of the working area. Similar relationships exist for other types of machine.

4.3.2 Limiting Influences Affecting the Measurement

For the characteristics to be reliably stated the operating and environmental conditions must be fully understood. These include the following data:

time and period of measurement;

ambient temperature (machine and measuring gauges must be exposed to this temperature for a sufficiently long period);

any other environmental conditions;

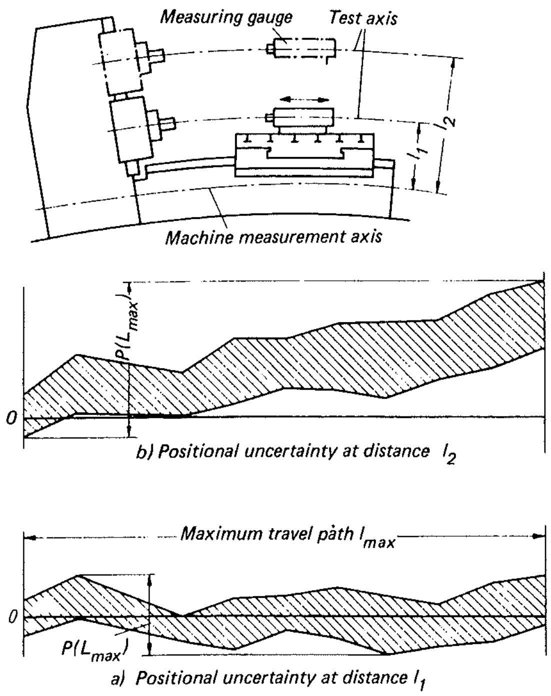


Fig. 10. Diagrammatic representation showing positional deviation is related to the situation which occurs when the machine measurement and test axes do not coincide

method of setting up (it should be noted that additional errors can occur when improvised setting up is used);

operating condition before measurement (limiting conditions such as spindle speed, slide feed rate and warming up time);

information about possible dead loading;

method of measurement and information about measurement uncertainty;

plan of the measuring arrangement with position of the working spindle, the machine measurement axes and the test axes (e.g. distances l_1 and l_2 in figure 10) and the positions of the measuring points;

description of the measuring procedure and information about travel conditions (travel time, speed of travel, slide positioning, path of travel).

The values of the factors affecting the results must be changed as little as possible during the test.

When comparing measurement results which have been obtained at different times, variations in the values of these factors must be taken into account: standard deviation s_j and therefore the positional scatter P_{s_j} are particularly sensitive to temperature drift.

4.4 Example of the Evaluation of the Characteristics

An example is given below of the determination of the characteristics in a chosen test axis. The desired value at each measurement position was preset on the machine controls. The individual values are the deviations from the desired value. They are recorded in table 1, pages 14/15. The standard deviation is in addition calculated by the range method. The characteristics are plotted graphically

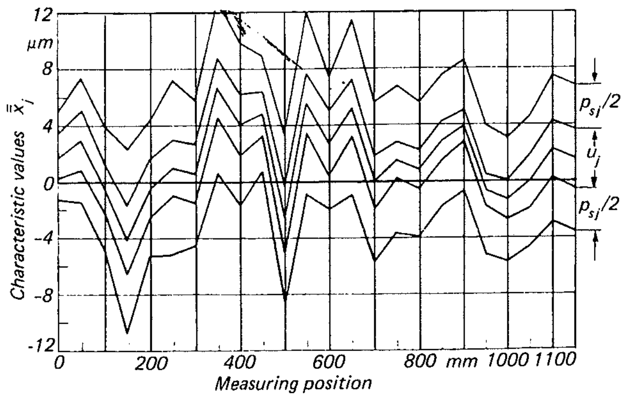


Fig. 11. Graphical representation of the characteristic values

at each measurement position within the chosen test axis as shown in Fig. 11.

This detailed example helps an understanding of the method of analysis. In practice it is advisable to undertake a computer analysis, recording and graphical presentation of the results in support of this.

5 Advice on the Choice of Test Methods and Test Characteristics

When testing the operating uncertainty of a machine tool by machining a number of components or test workpieces which are clamped over the whole working area of a machine, all the limiting factors attributable to the machine must be taken into account.

From the economic point of view it is not generally feasible to use test workpieces to such an extent that all systems deviations can be detected.

Only the random deviations, that is the operational scatter of a machine are therefore determined by this method, and possibly in particular cases the operational scatter as regards shape or any other entity (section 3.2.1.1). The chosen characteristic value set in relation to the workpiece machining tolerance (section 3.3) enables a decision to be made about the serviceability of a machine for a specific purpose.

The machining of test workpieces is of most significance where the dimensions of a workpiece are not the only criterion for assessment, but also other qualities such as deviations of shape or surface quality. In these cases the appropriate operational scatter can only be determined by machining workpieces.

Consequently the following characteristic values are expedient for the assessment of the operational scatter of a machine tool:

operational scatter A_s,

Operational scatter with reference to shape $A_{s\ shape}$.

In the case of general purpose machines and in particular with numerically controlled devices, the positioning of machine saddles, tables, spindle sleeves (or quills) etc. is a machine function which is expediently tested by determining the positional uncertainty.

Its determination has the advantage that not only deviations of the system are detected by direct measurement, but also random deviations.

The measuring place within the operating area of the machine can be easily changed. The whole of the range of travel of each axis is covered, and the number of measuring positions within the area of travel can easily be increased. The number of measured values can be increased while maintaining the measuring period constant by using automatic measuring methods, and on the other hand a possible trend can be detected by increasing the measuring period.

Measurement takes place along the line of each axis while the other axes are at rest. It provides in particular evidence of the quality of the measuring system, the drive and the geometry of the slideways. Information about the geometrical interaction of the axes cannot be obtained at this stage. This requires additional tests. The machine is not under load from the workpiece and the machining process with this procedure.

The following characteristics are required for an assessment of the positional accuracy:

positional deviation P_a

maximum positional scatter $P_{s_{max}}$

mean positional scatter $\overline{P}_s$

maximum reversal error U_{max}

mean reversal error $\overline{U}$

For a better understanding it is advisable to prepare a graphical presentation of the effective deviation in each measurement position in accordance with Fig. 7, section 4.

Specification of the *positional tolerance* T_p and agreement on ranges of tolerance for all the important test characteristics forms a valuable basis for good relations between user and manufacturer. When making such agreement the environmental conditions must be taken into account in each case as well as the most important limiting factors (sections 3 and 4).

The criteria for the selection of a test method are in the first place the type of machine (special purpose – general purpose), the type of control (numerical control, cam operated and manually operated), and the desired information. Using both methods (operational scatter and positional uncertainty) the reliability of the evidence exceeds the expected operational uncertainty of the machine.

Table 2 is intended to serve as a guide as to which methods are preferable in the case of different types of machine. Those tests marked with X should always be carried out. The sign – indicates that the test may be omitted.

In the case of machining centres all possibilities of testing are frequently utilised in order to obtain reliable results.

In the case of automatic machines with cam operation, repeatability of workpiece dimension is of paramount importance, and therefore testing with test workpieces is the best method.

Table 2. Choice of test method

Machines			Geometrical test to DIN 8601	Positional test Systems deviation	Random deviation	Testing operational scatter (test workpieces
Tied to a component (special purpose)	general		X	–	(X)	X
not tied to a component (general purpose)	manually operated	setting by scale	X	–	–	X
		digital display	X	X	X	X
		gauging machine	X	X	X	(X)*)
	automatic	cam control	X	–	(X)	X
		tracer control	X	–	(X)	X
	numerically controlled	machining centres	X	X	X	X
		other machines	X	X	X	(X)
		gauging machines	X	X	X	(X)*)
		drawing machines	X	X	X	X

*) Check with test standards

Explanation:

X recommended

(X) possible

– not applicable

Table 1. Example of the determination of characteristic values

	Random sample position on the travel path in mm										
	0	51,110	100	152,22	200	253,33	300	354,44	400	455,55	500
n	Measurement points *m*										
Individual measured values ↑	1	2	3	4	5	6	7	8	9	10	11
1	0	0	–3	–8	–4	–3	–3	+3	+1	+2	–7
2	+1	+2	–1	–5	–2	+1	0	+6	+4	+4	–4
3	0	+1	–3	–6	–2	–2	–1	+6	+2	+4	–4
4	0	0	–2	–7	–2	0	–2	+4	+1	+2	–6
5	0	+1	–2	–7	–3	–1	–2	+4	+1	+4	–5
$\Sigma x_i \uparrow$	+1	+4	–11	–33	–13	–5	–8	+23	+9	+16	–26
$\bar{x}_j \uparrow = \frac{1}{5} \Sigma x_i \uparrow$	+0,2	+0,8	–2,2	–6,6	–2,6	–1,0	–1,6	+4,6	+1,8	+3,2	–5,2
$R_j \uparrow = \lvert x_{i\,max} \uparrow - x_{i\,min} \uparrow \rvert$	1,0	2,0	2,0	3,0	2,0	4,0	3,0	3,0	3,0	2,0	3,0
Individual measured values ↓	1	2	3	4	5	6	7	8	9	10	11
1	+3	+4	+1	–3	+1	+2	+2	+7	+5	+6	–2
2	+4	+5	+1	–2	+1	+2	+2	+8	+5	+6	–1
3	+3	+5	+1	–3	+1	+3	+2	+9	+7	+7	0
4	+3	+6	+3	+1	+2	+5	+4	+10	+7	+6	+1
5	+4	+5	+1	–2	+3	+3	+3	+10	+7	+7	–1
$\Sigma x_i \downarrow$	+17	+25	+7	–9	+8	+15	+13	+44	+3,1	+3,2	–3
$\bar{x}_j \downarrow = \frac{1}{5} \Sigma x_i \downarrow$	+3,4	+5,0	+1,4	–1,8	+1,6	+3,0	+2,6	+8,8	+6,2	+6,4	–0,6
$R_j \downarrow = \lvert x_{i\,max} \downarrow - x_{i\,min} \downarrow \rvert$	1,0	2,0	2,0	4,0	2,0	3,0	2,0	3,0	2,0	1,0	3,0
$\bar{R}_j = \frac{1}{2} (R_j \uparrow + R_j \downarrow)$	1,0	2,0	2,0	3,5	2,0	3,5	2,5	3,0	2,5	1,5	3,0
$\bar{\bar{x}}_j = \frac{1}{2} (\bar{x}_j \uparrow + \bar{x}_j \downarrow)$	+1,8	+2,9	–0,4	–4,2	–0,5	+1,0	+0,5	+6,7	+4,0	+4,8	–2,9
$U_j = \lvert \bar{x}_j \uparrow - \bar{x}_j \downarrow \rvert$	3,2	4,2	3,6	4,8	4,2	4,0	4,2	4,2	4,4	3,2	4,6
$\bar{s}_{jR} = \frac{\bar{R}_j}{d_n}$	0,430	0,860	0,860	1,505	0,860	1,505	1,075	1,290	1,075	0,645	1,290
$s_j \uparrow = \sqrt{\frac{1}{4} \Sigma (x_{ij} \uparrow - \bar{x}_j \uparrow)^2}$	0,447	0,837	0,837	1,140	0,894	1,581	1,140	1,342	1,304	1,095	1,304
$s_j \downarrow = \sqrt{\frac{1}{4} \Sigma (x_{ij} \downarrow - \bar{x}_j \downarrow)^2}$	0,548	0,707	0,894	1,643	0,894	1,225	0,894	1,304	1,095	0,548	1,140
$\lvert \bar{s}_j \rvert = \frac{1}{2} (s_j \uparrow + s_j \downarrow)$	0,498	0,772	0,866	1,392	0,894	1,403	1,017	1,323	1,2	0,822	1,222
$P_{s_{jR}} = 6 \cdot \bar{s}_{jR}$	2,6	5,2	5,2	9,0	5,2	9,0	6,5	7,7	6,5	3,9	7,7
$P_{s_j} = 6 \cdot \lvert \bar{s}_j \rvert$	3,0	4,6	5,2	8,4	5,4	8,4	6,1	7,9	7,2	4,9	7,3
$\bar{\bar{x}}_j + \frac{1}{2} (U_j + P_{s_j})$	4,9	7,3	4,0	2,4	4,3	7,2	5,7	12,8	9,8	8,9	3,1
$\bar{\bar{x}}_j - \frac{1}{2} (U_j + P_{s_j})$	–1,3	–1,5	–4,8	–10,8	–5,3	–5,2	–4,7	+0,6	–1,8	+0,7	–8,9

$(P_{s\,max})_R = 9\ \mu m$ $U_{max} = 4{,}8\ \mu m$ $(\bar{P}_s)_R = 6{,}7\ \mu m$ $P = 12{,}8 + 10{,}8 = 23{,}6\ \mu m$

$P_{s\,max} = 8{,}6\ \mu m$ $\bar{U} = 3{,}7\ \mu m$ $\bar{P}_s = 6{,}5\ \mu m$

556,66	600	657,77	700	758,89	800	859,99	900	951,11	1000	1052,22	1100	1153,33	
12	13	14	15	16	17	18	19	20	21	22	23	24	
+ 1	−1	+ 1	−4	−2	−2	−1	0	−5	−5	−3	−2	−1	
+ 4	+ 1	+ 4	−1	+ 2	+ 1	+ 2	+ 4	−1	−2	−1	0	0	
+ 4	0	+ 3	−2	−1	−1	+ 2	+ 3	−1	−2	−2	+ 1	−1	
+ 4	+ 1	+ 5	−1	+ 2	0	+ 2	+ 3	−1	−2	−2	+ 1	−1	
+ 4	+ 1	+ 3	−2	0	−1	+ 2	+ 4	−1	−3	−2	+ 1	0	
+ 17	+ 2	+ 16	−10	+ 1	−3	+ 7	+ 14	−9	−14	−10	+ 1	−3	
+ 3,4	+ 0,4	+ 3,2	−2,0	+ 0,2	−0,6	+ 1,4	+ 2,8	−1,8	−2,8	−2,0	+ 0,2	−0,6	
3,0	2,0	4,0	3,0	4,0	3,0	3,0	4,0	4,0	3,0	2,0	3,0	1,0	
12	13	14	15	16	17	18	19	20	21	22	23	24	
+ 5	+ 4	+ 5	0	+ 3	+ 3	+ 5	+ 4	0	−1	0	+ 4	+ 1	
+ 8	+ 5	+ 7	+ 1	+ 2	+ 1	+ 4	+ 5	+ 1	0	+ 3	+ 5	+ 4	
+ 8	+ 6	+ 8	+ 2	+ 3	+ 1	+ 4	+ 5	0	0	+ 2	+ 4	+ 5	
+ 9	+ 5	+ 8	+ 3	+ 4	+ 3	+ 5	+ 6	0	+ 1	+ 2	+ 5	+ 4	
+ 8	+ 5	+ 8	+ 3	+ 2	+ 3	+ 3	+ 5	+ 1	0	+ 2	+ 3	+ 4	
+ 38	+ 25	+ 36	+ 9	+ 14	+ 11	+ 21	+ 25	+ 2	0	+ 9	+ 21	18,0	
+ 7,6	+ 5,0	+ 7,2	+ 1,8	+ 2,8	+ 2,2	+ 4,2	+ 5,0	+ 0,4	0	+ 1,8	+ 4,2	+ 3,6	
4,0	2,0	3,0	3,0	2,0	2,0	2,0	2,0	1,0	2,0	3,0	2,0	4,0	
3,5	2,0	3,5	3,0	3,0	2,5	2,5	3,0	2,5	2,5	2,5	2,5	2,5	
+ 5,5	+ 2,7	+ 5,2	−0,1	+ 1,5	+ 0,8	+ 2,8	+ 3,9	−0,7	−1,4	−0,1	+ 2,2	+ 1,5	
4,2	4,6	4,0	3,8	2,6	2,8	2,8	2,2	2,2	2,8	3,8	4,0	4,2	$\Sigma U_i = 88{,}6$
1,505	0,860	1,505	1,290	1,290	1,075	1,075	1,290	1,075	1,075	1,75	1,075	1,075	
1,342	0,894	1,483	1,225	1,789	1,140	1,342	1,643	1,789	1,304	0,707	1,304	0,548	
1,517	0,707	1,304	1,304	0,837	1,095	0,837	0,707	0,598	0,707	1,095	0,837	1,517	
1,430	0,801	1,394	1,265	1,313	1,118	1,090	1,175	1,168	1,005	0,901	1,071	1,033	
9,0	5,2	9,0	7,7	7,7	6,5	6,5	7,7	6,5	6,5	6,5	6,5	6,5	$\Sigma P_{s_j R} = 160{,}3$
8,6	4,8	8,4	7,6	7,9	6,7	6,5	7,1	7,0	6,0	5,4	6,4	6,2	$\Sigma P_{s_j} = 157{,}0$
11,9	7,4	11,4	5,6	6,8	5,6	7,5	8,6	3,9	3,0	4,5	7,4	6,7	
−0,9	−2,0	−1,0	−5,8	−3,8	−4,0	−1,9	−0,7	−5,3	−5,8	−4,7	−3,0	−3,7	

6 Appendix

Explanations of the most important statistical techniques and concepts

6.1 Parent Population, Distribution, Random Sampling

The total number of parts machined on a machine are designated *parent population.* Due to various influences the dimensions of the parts are scattered over a specific range. If the class frequency H_j is plotted on the scale (x) according to a suitable classification around the class mean x_j (j = 1, 2, 3, . . .), then we get a frequency *distribution* which, given uninterrupted machining, small scatter and a large number of parts, can be approximated by a symmetrical bell-shaped curve, Fig. 12. This normal (or Gaussian) distribution is uniquely defined by the mean value μ (as a measure of position) and the standard deviation σ (as a measure of scatter). In reality it is not possible for economic reasons to measure a very large number of parts. We therefore limit ourselves always to n random samples. For the mean value and standard deviation in this case the appropriate estimated values $\bar{x}$ and s are used.

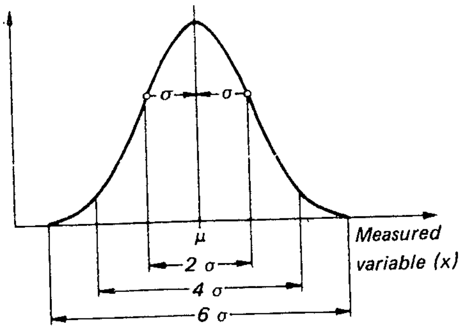


Fig. 12. Standard distribution

6.1.1 Mean Value

The *arithmetical mean* or *mean value* in short of a random sampling of n values $x_1, x_2, \dots x_n$ is given by:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \dots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_1$$

The mean value μ corresponds to this in a normally distributed totality; the bell-shaped curve rises to a maximum at the point where $\bar{x} = \mu$ (Fig. 12).

6.1.2 Standard Deviation

For normal distribution the *standard deviation* σ means the distance between the axis of symmetry and the point of inflection of the bell-shaped curve. It is a measure of the scatter of the measured values around the mean value (Fig. 12).

In the range between $\mu - \sigma$ and $\mu + \sigma$ lie

68.26 % of all measured values.

In the range between $\mu - 2\,\sigma$ and $\mu + 2\,\sigma$ lie

95.40 % of all measured values.

In the range between $\mu - 3\,\sigma$ and $\mu + 3\,\sigma$ lie

99.76 % of all measured values.

For determining the estimated value s for the standard deviation σ of the random sample the following formula is used:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_1 - \bar{x})^2}$$

6.2 Methods of Approximation for the Determination of Mean Value and Standard Deviation

Calculation of the approximate standard deviation s using the above formula is fairly time consuming. In practice therefore the following two methods of approximation are used. They are generally sufficiently accurate.

6.2.1 Graphical Method with the Aid of the Probability Network Diagram

For a better understanding, an example is re-worked alongside the formulas, Fig. 13.

From the measured values of the random samples the range R is first obtained as the difference between the maximum and minimum values:

$$R = x_{max} - x_{min}$$

This range is sub-divided into several equal sized intervals, the *classes.*

Reference values for a number k of classes are given by:

$$k \approx \sqrt{n}$$

In this formula n denotes the number of measured values. There must, however, be at least 5 classes, as otherwise a reliable result is not possible.

In the example $k = \sqrt{64 = 8.}$

The *range of classes* must be chosen so that each individual value can be assigned to a class. It is calculated from:

$$w = \frac{R}{k-1}$$

In the example R = 339 – 304 = 35 mm; from which the classification range becomes:

$$w = \frac{35}{8-1} = 5 \text{ mm}$$

In this way the following classifications are obtained:

> 301,5 to 306,5 mm
> 306,5 to 311,5 mm
> 311,5 to 316,5 mm
> 316,5 to 321,5 mm
> 321,5 to 326,5 mm
> 326,5 to 331,5 mm
> 331,5 to 336,5 mm
> 336,5 to 341,5 mm

The analysis sheet (Fig. 13) shows the frequency distribution in the upper half and in the lower half this distribution in the probability network.

The classes determined are recorded in sequence in the line marked "measured variable" and the measured values assigned to the individual classes with crosses. The frequency diagram in the example gives the approximate shape of the standard distribution.

The number of measured values in the respective classes (*class frequency*) is recorded in line "n_j".

In line "G_j" the number of measured values are added up successively from class to class (*summated class frequency*).

In line "H_j" the values for "G_j" are expressed as percentages. They represent the *cumulative frequency.*

$$H_j = \frac{G_j \cdot 100}{n} \quad (\%)$$

The values of the cumulative frequency are then plotted in the probability diagram.

The cumulative frequency in the first class is 3.1 %. The associated point in the probability diagram is found to be the point of inter-

Not for machine scatter, only for production scatter!

Note:

1) Series of measurements with 25, 50 or 100 measured values are preferred since then calculation of total–% does not apply (see left and right scales on the probability diagram)

2) Probability line: plot G_j or H_j on the probability line to right class limits. Connect points with a compensating line.

3) Mean value at 50%. 6s –range lies between the points of intersection of the compensating lines with the horizontal at 99.85% and 0.15%

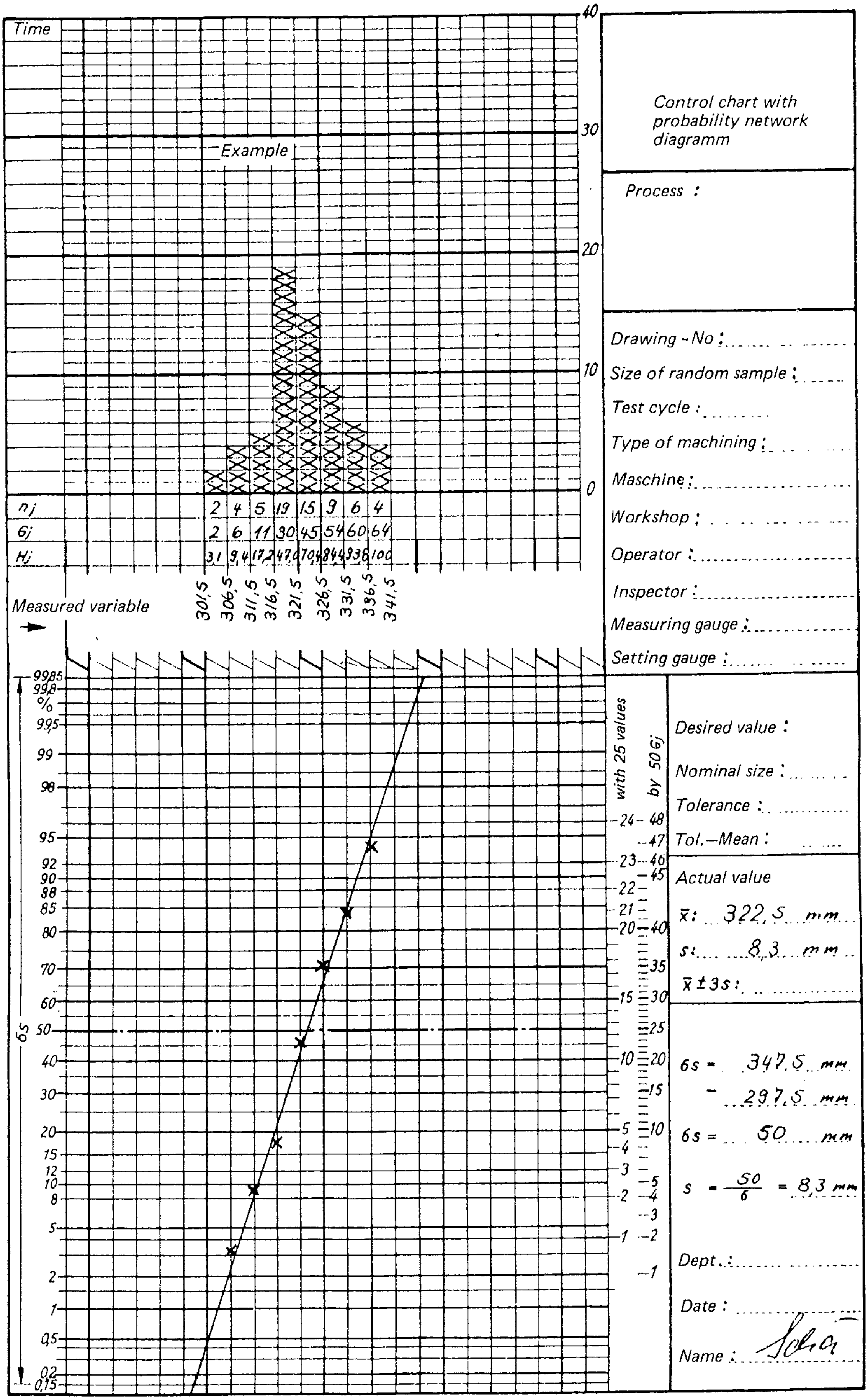


Fig. 13. Analysis with the probability network diagram

section of the vertical line 306.5 mm with the horizontal line 3.1 %. This applies appropriately to all cumulative frequencies of the individual classes. A compensative line is drawn through the points in the probability diagram.

The range of scatter 6_s lies in the area between the points of intersection of the compensating lines with the 0.15 % and 99.85 % lines.

The mean value $\bar{x}$ is given by the point of intersection of the curve with the 50 % line.

6.2.2 Range Method

The whole random sample is divided into m individual random samples of for example $n = 5$ parts. The range R (difference between the maximum and minimum value) is obtained from the measured values of 5 parts. The mean range is given as the arithmetical mean of the ranges of the m random samples

$$\bar{R} = \frac{\Sigma R}{m}$$

The standard deviation is obtained from the equation

$$s_R + \frac{\bar{R}}{d_n}$$

d_n in the above is a factor which is taken from the following table. It is dependent on the number of measured values in the individual random sample.

Number of values in the individual random sample	2	3	4	5	6	7
d_n =	1,128	1,693	2,059	2,326	2,534	2,704
Number of values in the individual random sample	8	9	10	12	16	20
d_n =	2,847	2,970	3,078	3,258	3,532	3,735

By this means the *mean value* is calculated as follows: from each random sample is obtained its arithmetical mean value $\bar{x}$.

The *total mean value* $\bar{\bar{x}}$ is then calculated from the m random sample mean values from

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{m}$$

6.3 Determination of the Confidence Range for Mean Value and Standard Deviation

The characteristic values $\bar{x}$ and s which are obtained from the statistical analysis of the measured values represent estimated values for the corresponding characteristic values μ and σ of the parent population. If a sufficiently large number of measured values ($n \geqslant 50$) are taken the x and s values are accurate enough. Further calculation for the confidence ranges is not necessary.

Since the confidence ranges for the values μ and σ depend on the size of random samples, quite considerable differences can occur between the random sample values $\bar{x}$ and s and the values of the parent population μ and σ in the case of smaller random samples however.

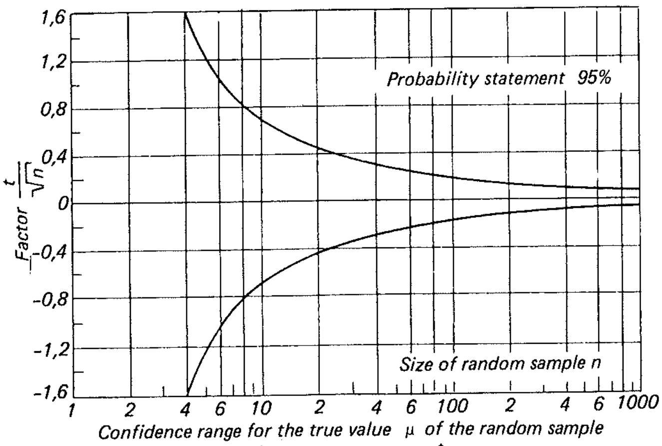
Note: When machining test workpieces m random samples are taken for each of n individual measurement values. The extent of random sampling $m \cdot n$ is then taken as the basis for calculating the confidence range for $\bar{x}$ and s. In calculating the positional characteristic values per random sampling position [n (pos) and n (neg)] are determined. The confidence range for $\bar{x}_j$ and s_j refers in this case to the random sampling of 2 n.

For the practical determination of the positional characteristic values the random sample values $\bar{x}$ and s for a random sampling size of $n = 10$ are generally sufficient, if there are 6_s-dispersion areas.

In special cases where the possible deviations between random sampling characteristic values and the characteristics of the parent population must be accurately assessed, the determination of the confidence range is summarised below:

For the mean value:

With 95 % probability statement the mean value μ of the parent population lies in the area $\bar{x} \pm \frac{t}{\sqrt{n}} \cdot s$. The factor $\frac{t}{\sqrt{n}}$ is taken from Fig. 14.



$$\bar{x} - \frac{t}{\sqrt{n}} \cdot s_R \leqq \mu \leqq \bar{x} + \frac{t}{\sqrt{n}} \cdot s_R$$

Fig. 14. Confidence range for the mean value

The confidence range for the true mean value μ_j fixes the absolute location of position x_j.

For the standard deviation:

With 95 % probability statement the standard deviation of the parent population lies between

$$1/D_{up} \cdot s_R \text{ and } 1/D_{low} \cdot s_R$$

The following applies

$$1/D_{up} \cdot s_R \leqslant \sigma \leqslant 1/D_{low} \cdot s_R$$

The factors $1/D$ for the standard deviations s_R determined by the range method are taken from Fig. 15. To determine the confidence ranges of μ and σ of the parent population, when s has been determined according to the exact method of calculation, the values given in Fig. 14 and 15 for the range method can be used in practice with sufficient accuracy.

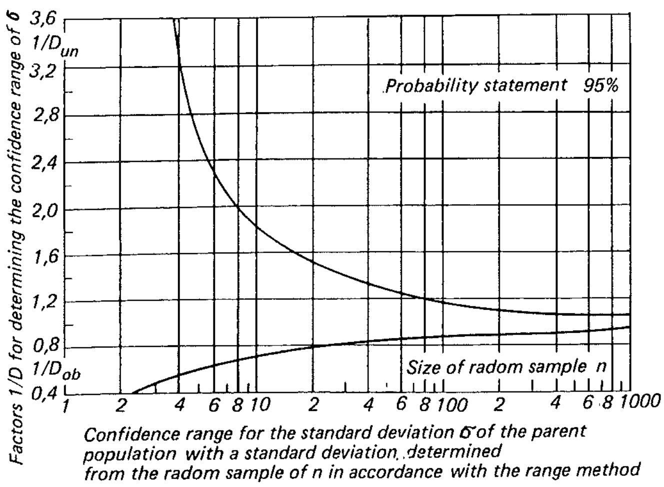


Fig. 15. Confidence range for the standard deviation

6.4 Determination of the Measurement Uncertainty of the Measuring Gauges Used

Special attention should be given to the accuracy of the measuring gauges used, since the results of a statistical test can be strongly falsified if the uncertainty of the measurement procedure is too great.

The scatter of the measuring gauges to be used must therefore be investigated prior to every test, unless it is already known.

The measurement uncertainty must as far as possible not exceed 10 % or at the most however 20 % of the specified machining tolerance when the statement probability is 95.4 % ($4\,s_{R\ equipt.}$).

If the measurement uncertainty of the method used is considered with reference to the expected or determined operational scatter A_s, then the following relationship should exist: $4\,s_{R\ equipt.} \leq 0.25$ to $0.3\,A_s$. This relationship is obtained from the reference values recommended in section 3.3.

For the practical determination of measurement uncertainty the random sample should consist of at least 24 parts. The parts are numbered and measured twice, one after the other, in the same order and each time at exactly the same measuring point with the measuring gauges used for the test. Determination of measurement uncertainty is explained by an example below.

6.4.1 Example for Determining the Measurement Uncertainty of a Measuring Gauge

Determination of measurement uncertainty is explained by the example of a measuring device consisting of measuring stand, surface table and dial indicator. The diameters of the 24 cylindrical parts are measured. The deviations from the set dimension taken from two successive measurements at a specified measuring point are given in table 3.

The 24 measured values are sub-divided into three groups each containing eight values. The difference between the results of the 1st and 2nd measurements are then taken (make sure which is positive and which negative!).

The range R is now worked out for each group from the three R values of the arithmetical mean range value $\bar{R}$. The scatter of the dimensional difference is first obtained by the range method and then the standard deviation of the measurement uncertainty of the equipment $s_{R\ equipt.}$ after dividing by 2. By total measurement uncertainty is meant the area $4\,s_{R\ equipt.}$ (95 % statement probability).

If the area $4\,s_{R\ equipt.}$ of measurement uncertainty is greater than 20 % of the workpiece tolerance, more accurate measuring

Table 3. Deviations from the set dimension taken from two successive measurements

Test specimen No.	1st measurement µm	2nd measurement µm	Difference Δ µm	Range R_Δ µm
1	14,6	13,7	+ 0,9	2,3
2	20,8	19,6	+ 1,2	
3	11,6	10,9	+ 0,7	
4	19,1	19,0	+ 0,1	
5	24,1	22,0	+ 2,1	
6	12,0	12,0	0	
7	19,3	18,5	+ 0,8	
8	14,2	14,4	–0,2	
9	13,5	12,0	+ 1,5	1,8
10	17,8	18,1	–0,3	
11	12,6	11,5	+ 1,1	
12	11,7	10,6	+ 1,1	
13	12,4	12,7	–0,3	
14	17,0	15,7	+ 1,3	
15	15,3	14,1	+ 1,2	
16	13,8	14,1	–0,3	
17	23,5	26,1	–2,6	5,2
18	19,4	18,4	+ 1,0	
19	15,5	14,8	+ 0,7	
20	14,6	12,0	+ 2,6	
21	15,0	15,8	–0,8	
22	16,2	17,3	–1,1	
23	14,8	13,8	+ 1,0	
24	16,7	17,0	–0,3	

equipment must be used, or, if this is not possible, the operational scatter must be reduced by the measurement uncertainty in accordance with the following analysis of the results of the investigation.

$$s_R = \sqrt{s_R^{*2} - s_{R\ equipt.}}$$

s_R the operational scatter reduced by the measurement uncertainty

s_R^* scatter calculated from the measured values

$s_{R\ equipt.}$ scatter of the measurement uncertainty

$\Sigma R_\Delta = 9.3$

$$\bar{R}_\Delta = \frac{\Sigma R_\Delta}{3} = \frac{9.3}{3} = 3{,}1\ \mu m$$

$$s_{R\Delta} = \frac{\bar{R}_\Delta}{d_n} = \frac{3.1}{2.85} = 1.08\ \mu m$$

$$s_{R\ equipt.} = \frac{s_{R\Delta}}{2} = \frac{1.08}{2} = 0.76\ \mu m$$

If it is found during an investigation that the measurement uncertainty of the measuring equipment is too great, the cause for the error must always be sought, or new equipment or another method used.

6.5 Trend

The cause of trend is generally tool wear, which leads to system dimensional deviation. Other influences such as temperature change can, however, give rise to trend phenomena.

6.5.1 Determination of Trend

Trend can most simply be determined graphically (section 3.5). For this purpose the measured values are plotted against the workpiece numbers which are plotted in chronological order. If this is too time consuming due to a very large number of measured values, then instead of plotting the individual measured values x, the mean values $\bar{x}$ of the individual random samples m are plotted. A compensating line is drawn through the plotted points. In many cases the compensating curve is a straight line. If no linear relationship exists, the curve can be approximated piecemeal by straight lines.

With graphical representation of all measured values Δx is the greatest ordinate difference of the compensating line.

$$T_N = \Delta x$$

If, however, the mean values $\bar{x}$ of the individual random samples are plotted, the compensating line must be extended by the distance of one random sample to determine T_N, in order to obtain the corresponding ordinate difference.

This procedure is generally sufficiently accurate. Although a somewhat more accurate estimate of the trend can be obtained by calculation, it is more time consuming however.

6.5.2 The Effect of Trend on the Standard Deviation

The operational scatter is calculated from the R values of the individual random samples using the range method. The range is composed of the actual scatter of the machine and a share given by the steepness of the trend lines, caused for instance by toolwear. Consequently, too great a standard deviation s_R is obtained with too great a trend by the range method. Since the trend cannot always be attributed to the machine, account must be taken of the operational scatter when making the evaluation.

The operational scatter can be corrected in two different ways.

6.5.2.1 Graphical Correction of the Operational Scatter A_s

The individual values of the random sample are plotted graphically in the order of machining, Fig. 16.

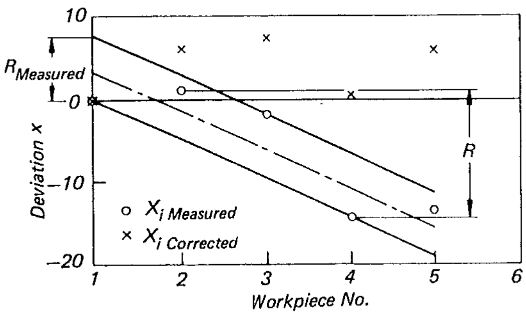


Fig. 16. Graphical determination of the effect of trend on the range R

An averaged line, the trend line, is drawn through the measured values. Two lines are drawn parallel to these trend lines, running through the measured values which are furthest apart from the trend lines. The corrected range R_{corr} is now obtained from the ordinate as the distance between the two lines. From this is obtained the corrected operational scatter

$$A_s = 6 s_R = 6 \cdot \frac{R_{corr}}{d_n}$$

Similarly when evaluating several random samples

$$A_s = 6 \frac{\bar{R}_{corr}}{d_n} ; \; \bar{R}_{corr} = \frac{1}{m} \Sigma R_{j\,corr}$$

6.5.2.2 Correction of Operational Scatter A_s by Calculation

Using as an example the same measured values as are plotted in Fig. 16, the calculation method is demonstrated.

Even when using the calculation method, the trend T_N is determined graphically against the number of machined workpieces for each random sample. From this the trend for each workpiece is calculated as follows:

$$T_S = \frac{T_N}{n-1} \quad \text{(where } N = n = 5 \text{ in the example)}$$

The individual measured values include a share of the trend as shown in the following tabulation:

Workpiece number	1	2	3	... n
Share of trend x_{iT}	$(1-1)\,T_S$	$(2-1)\,T_S$	$(3-1)\,T_S$	... $(n-1)T_S$

The trend share of the individual workpiece

$$x_{iT} = (i-1)\,T_S$$

can be used to correct each measured value $x_{i\,meas}$.

$$x_{i\,corr} = x_{i\,meas} - x_{iT}$$

The corrected range becomes

$$R_{corr} = |x_{i\,corr\,max} - x_{i\,corr\,min}|$$

Example of the calculation: values in µm

from figure 16 $T_N = -19$ µm

and $T_S = -19/4 = -4{,}75$ µm

Workpiece number	1	2	3	4	5
$x_{i\,meas}$	0	+ 1	– 2	– 14	– 13
x_{iT}	0	– 4,75	– 9,5	– 14,25	– 19
$x_{i\,corr}$	0	+ 5,75	+ 7,5	+ 0,25	+ 6

The uncorrected range results in

$$R = x_{2\,meas} - x_{4\,meas} = 15 \; \mu m$$

On the other hand the corrected range results in

$$R_{corr} = x_{3\,corr} = 7.5 \; \mu m$$

Using the corrected range R_{corr} the standard deviation $s_{R\,corr}$ and from it the corrected operational scatter $A_s = 6\,s_R$ can be determined in accordance with section 6.2.2.

Similarly when evaluating several random samples

$$A_s = 6 \frac{\bar{R}_{corr}}{d_n}$$

7 List of Symbols Used

A_s	Operational scatter
d_n	Factor dependent on the number of measured values of each random sample
D_{ob}, D_{un}	Factor for calculating the scatter limits of s_R (In the translation these have been rendered D_{up} and D_{low})

f	Relative operational scatter
G_j	Summated class frequency
H_j	Frequency total
i	Index; ordinate number of the individual values in a random sample
j	Index; ordinate number of the random samples
K_s	Factor
k	Number of classes
L	Length of travel
L_{max}	Max. travel path
L_0	Reference point within the area of travel
l	Distance between test axis and machine axis
m	Number of the random sample or measuring position
N	Total number of measured values
n	Number of parts per random sample or positioning procedure for each measured position
n_j	Class frequency of the j^{th} class
P	Positional uncertainty
P_a	Positional deviation
P_s	Positional scatter
$\bar{P}_s$	Mean positional scatter
P_{s_j}	Positional scatter at point x_j
$P_{s_{max}}$	Max. positional scatter
R	Range
$\bar{R}$	Mean range
R_j	Range R in the random sample j or at point x_j
$\bar{R}_j$	Mean range at point x_j
$\bar{R}_{korr}$	Mean corrected range (Note: in the translation this has been rendered as $\bar{R}_{corr}$)
s	Standard deviation
$\bar{s}$	Mean standard deviation
s_j	Standard deviation of the individual values in the random sample j
s_R	Standard deviation in accordance with the range method
$\bar{s}_R$	Mean standard deviation in accordance with the range method
s_R^*	Standard deviation modified for the influence of trend
s_R^{**}	Standard deviation of the measured values
$\bar{s}_{jR}$	Mean standard deviation of the individual values at point x_j in accordance with the range method
$s_{R\,Gerät}$	Standard deviation of the measuring uncertainty (*Note*: in the translation this has been rendered as $s_{R\,equipt.}$)
T	Machining tolerance
T_N	Trend
T_p	Positional tolerance
T_S	Trend for each individual value
U	Reversal error
$\bar{U}$	Mean reversal error
U_j	Reversal error at point x_j
U_{max}	Maximum reversal error
$U_{j\,max}$	Max. reversal error at point x_j
x_{iT}	Trend share within the individual value
$\bar{x}$	Mean value of the individual values
$\bar{\bar{x}}$	Total mean value of the individual values
x_j	Random sampling position
$\bar{x}_j$	Mean value of the individual values in the random sample j or at point x_j
$\bar{\bar{x}}_j$	System deviation from the desired value at point x_j
x_{ij}	Individual value i in the random sample j or at point x_j
$x_{i\,gem}$	Measured value (*Note*: in the translation this has been rendered as $x_{i\,meas}$)
x_{korr}	Corrected measured value (*Note*: in the translation this has been rendered as x_{corr})
w	Class interval
μ	Theoretical mean value ($n \rightarrow \infty$)
σ	Theoretical standard deviation ($n \rightarrow \infty$)
$\uparrow$	Positive direction of travel
$\downarrow$	Negative direction of travel

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